D.B.F.Dayanand College of Arts and Science, Solapur
Department of Mathematics
National Mathematics Day -2019

Date : 23/12/2019

DR.M.K.KUBDE MATHEMATICS QUIZ COMPETITION 2019

RESULT

Group Number	Score	Group Number	Score
G-1	**22**	G-18	9
G-2	**29**	G-19	18
G-3	10	G-21	12
G-4	7	**G-22**	**25**
G-5	2	G-23	11
G-6	5	G-24	8
G-7	3	G-25	12
G-8	6	**G-26**	**21**
G-9	3	G-27	19
G-10	5	G-28	4
G-11	**18**	G-29	13
G-12	16	**G-30**	**41**
G-13	11	G-31	11
G-14	5	G-32	7
G-16	17		

Congratulations to all the participants and winners!

Rank	Level-I	Level-II
1st	G-2 ,Score 29	G-30 ,Score 41
2nd	G-1 ,Score 22	G-22 ,Score 25
3rd	G-11 ,Score 18	G-26 ,Score 21

Instructions :

1. Prize distribution will be held on **Tuesday, 24th December 2019 at 03:30 pm**. All the members of the winning teams are requested to remain present for the function.
2. The decision of the organizers is final and no objections are entertained.
3. For any other queries, please contact Mr.A.R.Reshimkar Mobile : 8788983047.

Convener

23/12/2019

Head

Department of Mathematics

Head,
Department of Mathematics,
D.B.F. Dayanand College of Arts and Science, Solapur.

Dr.M.K.KUBDE MATHEMATICS QUIZ COMPETITION-2019

Organized
by
Department of Mathematics
D.B.F.Dayanand College of Arts and Science,Solapur

Day and Date : Friday,20th December 2019

Group Number :............... Level : 1 Marks : 100 Duration : 2 Hrs

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Instruction : Each question carries 5 Marks.

Part-I : ALGEBRA

Q 1. If a, b, c are roots of the polynomial $x^3 - 3x^2 + 2x + 5$ then compute $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.
Q 2. Find the area of a figure obtained by joining all 4^{th} roots of unity.
Q 3. If G_1, G_2 are two groups then prove that $G_1 \times G_2 \cong G_2 \times G_1$.
Q 4. If H is a subgroup of G then prove that $N(H)$ is the largest subgroup of G in which H is normal.
Q 5.Let $G = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \mid a \neq 0, b \neq 0 \right\}$ be multiplicative group then prove that $< \mathbb{C}^*, \times >$ is isomorphic to $< G, \times >$.

Part-II : MATRICES

Q 6. Obtain the solution space of the matrix equation $AX = 0$ for $A = \begin{bmatrix} 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 0 \end{bmatrix}$.
Q 7. If $x \begin{bmatrix} 3 \\ 5 \\ -4 \end{bmatrix} + y \begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix} + z \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$,then find values of x, y, z.
Q 8. If A is a square matrix and λ is an eigen value of A and X is a corresponding eigen vector,then prove that for each $n \in \mathbb{N}$,prove that λ^n is eigen value of A^n and X is the corresponding eigen vector.
Q 9. Let V be a vector space of all polynomials in one variable with real coefficients and of degree less than or equal to 2. Define the linear transformation
$T(a + bx + cx^2) = a + b(1 + x) + c(1 + x)^2$. Obtain the matrix representation of the linear transformation with respect to basis $\{1, 1 + x, 1 + x^2\}$.

Part-III : REAL ANALYSIS

Q 10.If $f : X \to Y$ is a function and if $B_1, B_2 \subset Y$ then prove that

i) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$
ii)$f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$
Q 11. Let $f : \mathbb{R} \to \mathbb{R}$ be continuously differentiable. Evaluate

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} f'\left(\frac{k}{n}\right)$$

Q 12. Show that $f(x) = x|x|$ is differentiable at $x = 0$.
Q 13. Prove that if $x > -1$ then prove that $(1 + x)^n \geq 1 + nx \ \forall \ n \in \mathbb{N}$. This is known as Bernoulli's Inequality.
Q 14. Prove that every point of an open interval is its limit point.

Part-IV: COMPLEX ANALYSIS

Q 15. Let C be the contour in the complex plane consisting of two straight line segments,one from $z = 0$ to $z = i$ and the other from $z = i$ to $z = 1 + i$. Let $f(z) = y - x - 3x^2 i, z = x + iy, x, y \in \mathbb{R}$. Evaluate $\int_C f(z)dz$.
Q 16. Evaluate : $\int_{\{|z|=2\}} \frac{dz}{(z-1)^3}$.

Part-V: CALCULUS

Q 17. Find the particular integral of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x} sinx$.

Q 18. Evaluate : $\int_0^1 max\left\{x, \frac{1}{2} - x\right\}dx$
.
Q 19. Evaluate :If $y(x) = \int_{\sqrt{x}}^{x} \frac{e^t}{t} dt, x > 0$ then find $y'(1)$.
Q 20. Evaluate :

$$lim_{(x,y)\to(2,-2)} \frac{\sqrt{x-y} - 2}{x - y - 4}$$

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Dr.M.K.KUBDE MATHEMATICS QUIZ COMPETITION-2019

Organized
by
Department of Mathematics
D.B.F.Dayanand College of Arts and Science,Solapur

Day and Date : Friday,20th December 2019

Group Number :...............	Level : 2	Marks : 100	Duration : 2 Hrs

Instruction : Each question carries 5 Marks.

Part-I : ALGEBRA

Q 1. If G_1, G_2 are two groups then prove that $\dfrac{G_1 \times G_2}{G_1 \times \{e_2\}} \cong G_2$ and $\dfrac{G_1 \times G_2}{\{e_1\} \times G_2} \cong G_1$.

Q 2. Prove that $\dfrac{GL(n,\mathbb{R})}{SL(n,\mathbb{R})} \cong \mathbb{Z}_2$.

Q 3. Prove that the group G of all real matrices of the form $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ is a subgroup of $GL(2,\mathbb{R})$ and it is isomorphic to a set of all mappings $\alpha_{a,b} : \mathbb{R} \to \mathbb{R}$ given by $\alpha_{a,b}(x) = ax + b, x \in \mathbb{R}$.

Q 4. If N is normal subgroup and H is a subgroup of a group G then prove that NH is a subgroup of G.

Q 5.Show that in a ring $\mathbb{Z} \times \mathbb{Z}$,prove that the ideal $S = \{(a, 0) | a \in \mathbb{Z}\}$ is a prime ideal but not a maximal ideal.

Q 6. Show that $J = \{f(x) \in \mathbb{Z}[x] \ : \ 5 \mid f(0)\}$ is an ideal of $\mathbb{Z}[x]$.

Q 7. For any group G and its subgroup H,prove that $x^{-1}N(H)x = N(x^{-1}Hx)$,for $x \in G$.

Part-II : REAL ANALYSIS

Q 8.Let $f : X \to Y$ be a function and $A \subset X, B \subset Y$. Then prove that

i. $f(f^{-1}(B)) \subset B$ ii. $A \subseteq f^{-1}(f(A))$.

Q 9. Evaluate :

$$\lim_{n\to\infty} n\left[\sum_{j=1}^{k} f\left(a + \frac{j}{n}\right) - kf(a)\right]$$

where $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function at $x = a$.

Q 10.Evaluate :

$$\sum_{k=1}^{\infty} \frac{k^2}{k!}$$

Part-III : COMPLEX ANALYSIS

Q 11. Let C be the contour consisting of the lines $x = \pm 2, y = \pm 2$ described counterclockwise in the plane. Compute $\int_C \bar{z} dz$.

Q 12. Find the residues at each of the poles for the function $f(z) = \dfrac{5z - 2}{z(z-1)}$.

Q 13. Write down power series expansion of $f(z) = \dfrac{1}{z^2}$ in the neighbourhood of $z = -1$.

Part-IV : MATRICES AND LINEAR ALGEBRA

Q 14. Find the inverse in $\mathbb{Z}_5$ of the following matrix : $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$.

Q 15. Let $\mathcal{P}_3(\mathbb{R})$ denote the vector space of all polynomials of degree at most 3 and $\{1, x, x^2, x^3\}$ be its standard basis. Define $T : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$ by $T(p) = p" - 2p' + p, p \in \mathcal{P}_3(\mathbb{R})$.Find the matrix representation of T with respect to the given basis.

Q 16.Let W_1, W_2 be two subspaces of a vector space V. Prove that V is the direct sum of W_1 and W_2 iff each vector in V can be uniquely written as $w_1 + w_2$,where $w_1 \in W_1$ and $w_2 \in W_2$.

Q 17. Let $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ defined by $T(X) = AXB$ where $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$.Find the trace and determinant of T.

Part-V : CALCULUS and DIFFERENTIAL EQUATIONS

Q 18. Evaluate : $\int_0^1 max\left\{x, \dfrac{1}{2} - x\right\} dx$

Q 19. For which real number $\alpha > 0$ does the differential equation $\dfrac{dx}{dt} = x^\alpha, x(0) = 0$ have a solution on some interval $[0, b], b > 0$.

Q 20. Let $y(x)$ be a solution of $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = 0, y(0) = 1, y'(0) = -1$,then find maximum value of $y(x)$ and the point of maximum.