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Subject	Physics
Paper	General Meteorology
Module No. & Title	1: Coriolis Force
Module Tag	DAYA_MET_EKK_M1

Objectives:

1. To know Different Frames of Reference and Pseudo Forces
2. To study Effects of Coriolis Force in Nature

Outcomes:

1. Understand Equation of motion with horizontal as well as vertical forces.
2. Understand when and how Coriolis force arises.

Module No. & Title	**1: Coriolis Force**
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Contents	**1. Introduction** **2. Non-Inertial Frame of Reference and Pseudo forces** **3. Coriolis Force** **4. Effects of Coriolis Force in Nature** **5. Summary**

1. Introduction

For a particle in motion the physics problem is to find the equations which determines the acceleration of the particle given as:

$$a = \frac{du}{dt} = Lim_{t\to 0}\frac{\Delta u}{\Delta t} = Lim_{t\to 0}\frac{u(t+\Delta t) - u(t)}{\Delta t} \qquad \dots (1)$$

By Newtons first law a body without forces is in uniform motion along a straight line. In Eqn. (1), $u(t+\Delta t) - u(t) = u(t)$ and at all the times the acceleration is zero.

The Newtons second law expresses mass into acceleration of a particle as the sum of all the forces acting on it, assuming a coordinate system fixed in space and tied to the stars. Let's apply similar idea to a parcel of air, with volume dV. If ρ is density of air then the mass of air of volume dV may be written as ρdV. The force which, for convenience we write, as mass into acceleration and we write

$$\rho dV\frac{du}{dt} = F_{total} = F_{\text{gravity}} + F_{\text{pressure}} + F_{Coriolis} + F_{Friction} \qquad \dots (2)$$

In Eqn. (2) the F_{total} includes, both vertical as well as horizontal forces. The first force on right hand side of Eqn. (2) is gravity force, acts vertically. The main effect of gravity force is in the hydrostatic equation

$$dp = -\rho g dz \qquad \dots (3)$$

The negative sign shows that the pressure decreases with increase in altitude, with $dz > 0$. But the vertical accelerations in air are generally small and we neglect the vertical component force in most cases. The remaining forces in Eqn. (2) are all horizontal, so horizontal components of Newtons law in Eqn. (2) are:

$$\rho dV\frac{du}{dt} = F_{total,horizontal} = F_{\text{pressure,horizontal}} + F_{Coriolis,h} + F_{Friction,h} \qquad \dots (2)$$

The Earth moves through space around the sun in an elliptical orbit with sun at one of foci and at the same time it rotates around its axis daily. For all purposes here we will ignore the annual motion and will consider daily motion only. Now our emphasis will be on the Coriolis force, before which we shall throw some light on Frames of Reference.

2. Non-Inertial Frame of Reference and Pseudo forces

Many problems in physics can be readily solved using moving coordinate system. In such problems two coordinate systems or frames of reference are considered. Out of two coordinate systems one is fixed and other is moving. Any inertial frame of reference used for observation is called fixed frame of reference. And the moving frame of reference can have translational or rotational motion or both simultaneously with respect to a fixed frame of reference.

A frame of reference which is at rest ($\vec{v} = 0$) or moving with a constant velocity ($\vec{v} = constant$) with respect to a fixed frame is called an inertial frame of reference. And a frame of reference accelerated relative to a fixed frame is called a non-inertial. All frames of reference rotating with respect to the fixed frame are non-inertial because, rotational motion is an accelerated motion.

The pseudo forces which arise in case of non-inertial frames of reference are centrifugal force and Coriolis force and known as non-inertial forces or pseudo forces or fictitious forces.

Newton's laws hold good in inertial frame of reference. In non-inertial frame of reference, the Newton's equation of motion ($\vec{F} = m\ddot{\vec{r}}$) is written as:

$$\text{Effective force} = \text{Real force } + \text{Non} - \text{inertial forces}$$

$$\vec{F}_{\text{eff}} = \vec{F}_{\text{real}} + \vec{F}_{\text{non-inertial}}$$

The $\vec{F}_{\text{real}}$ is a real or true force such as gravitational force, electrostatic force, magnetic force. An observer in the accelerated frame will measure the resultant or total force which is the sum of real and fictitious forces on the particle. The term $\vec{F}_{\text{non-inertial}}$ doesn't represent any real force. The real force has its existence due to certain kind of field of force or interaction and depends upon position and motion of other bodies.

The term $\vec{F}_{\text{non-inertial}}$ cannot be explained in any of the terms such as field of force or interaction so, it is called as fictitious or pseudo or non-inertial force. The non-inertial force has existence only in the non-inertial frame of reference. The pseudo force arises due to acceleration of the frame of reference.

e. g.,

i) Centrifugal force $= -mr\omega^2 = -m\,\vec{\omega} \times (\vec{\omega} \times \vec{r})$

ii) Coriolis force $= -2m(\vec{\omega} \times \vec{v}) = -2m\omega v \sin\phi$

The force which is not real and appears due to acceleration of the frame of reference is known as fictitious or pseudo force or non-inertial force.

The force which doesn't really act on the particle but appears due to the acceleration of the frame is called a fictitious or pseudo force.

Centrifugal force:

Centrifugal force is a fictitious force and is directed away from the centre along the radius. Remember that centrifugal force is a pseudo force and appears in the rotating or non-inertial frame of reference. The vector $\vec{\omega}$ should be shown making an angle of 23.5 ° with the vertical. It is shown vertical for sake of simplicity or convenience.

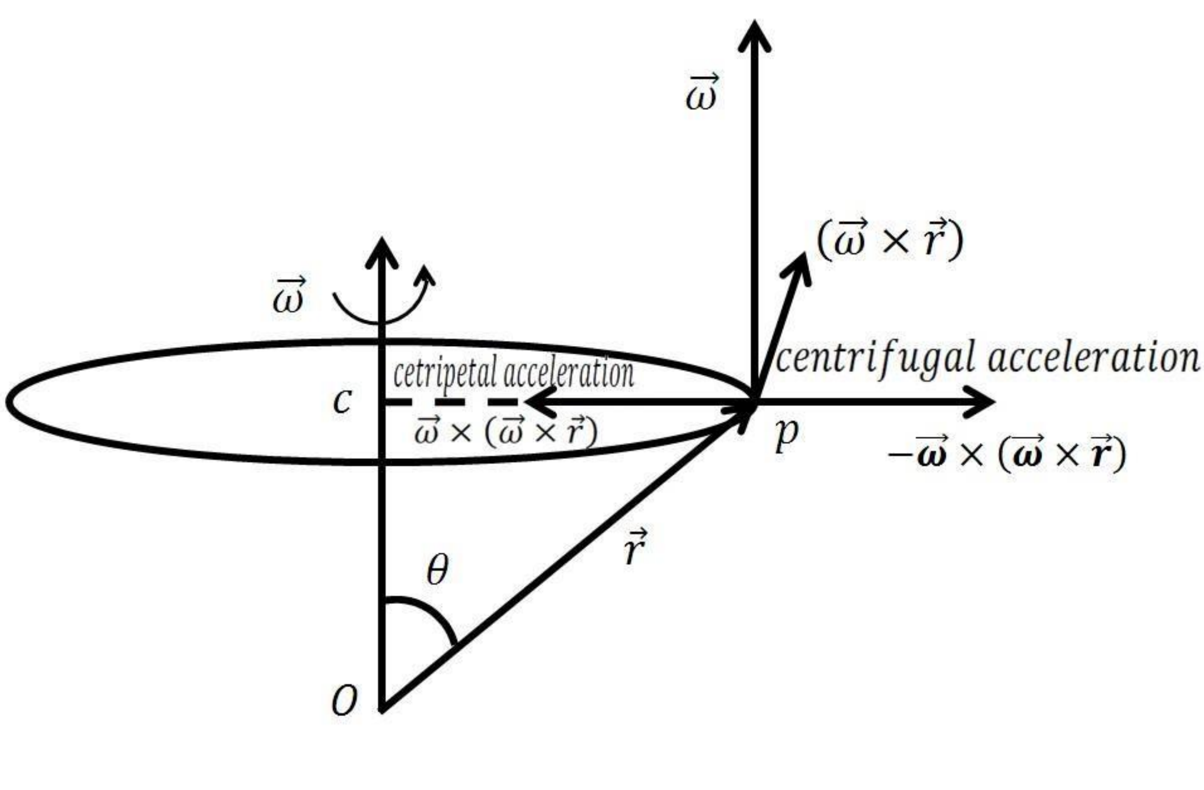

Fig. 1: Centrifugal force $(= -m\vec{\omega} \times (\vec{\omega} \times \vec{r}))$

For example,

1) When a bus takes turn on a curved road the passengers lean outward due to centrifugal force.
2) In a merry-go-round the persons experience outward pull due to centrifugal force.

Where is centrifugal force used?

1) Centrifuge machine in chemistry laboratory uses circular motion to segregate the chemicals.
2) Centrifuge machine in sugar industries use circular motion to filter the industrial water.

Effects: Earth is bulged at equator and flattened at the poles.

This centrifugal force is experienced during such circular motion and it's given by:

$$\text{centrifugal force} = -mr\omega^2 \quad = -m\,\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

In which r is position vector of the body or air mass.

Fixed or localized vector: A vector having tail at a fixed point is called a fixed or localized vector, e. g. position vector of a moving point is a fixed vector because its tail is fixed at origin.

Coriolis force:

Another fictitious force is Coriolis force. Coriolis force acts on a particle only if it is moving with respect to the rotating coordinate system. If a body is in motion in non-inertial frame of reference then it experiences Coriolis force. Coriolis force is a fictitious or pseudo or non-inertial force.

Thus, in a rotating frame of reference, if a body of mass m is moving with velocity v then the Coriolis force acting on a particle is given by:

$$\text{Coriolis force} = -2m\,(\vec{\omega} \times \vec{v}) = -2m\,\omega \mathrm{v}\,\sin\phi$$

In which $\vec{\omega}$ is angular velocity of the rotating frame of reference or angular velocity of spinning earth and ϕ is latitude of the place. The moving air mass always experiences a force perpendicular to its path opposite to the direction of vector product $(\vec{\omega} \times \vec{v})$.

Note that if the body is at rest $(\vec{v} = 0)$ in rotating coordinate system then no Coriolis force acts on it. If the body is at rest $\mathrm{v} = 0$ in rotating coordinate system then Coriolis force acting on it is zero. Further Corilolis force is zero at the equator; at $\phi = 0$.

And Coriolis force is maximum on either pole (on North Pole as well as South Pole).

Coriolis force acts at right angles to the direction of motion of air (or wind) to the right in north hemisphere and to the left in south hemisphere.
In north hemisphere, the Coriolis force acts perpendicular to the direction of motion of air to the right.

The Coriolis force acts at right angles to the direction of motion to the left in south hemisphere.

What are effects of Coriolis force in nature?

Coriolis force causes formation of cyclones,

deflection of missiles,

deflection of river flow and

deflection of freely falling body.

3. Coriolis Force

Coriolis force is experienced by a moving body in rotating frame of reference. Or if a body moves in rotating coordinate system then it experiences Coriolis force. Coriolis force is a pseudo force or fictitious force. The pseudo forces arise in non-inertial frames of reference.

$$\text{The Coriolis force } = -2m(\vec{\omega} \times \vec{V}) = -2m\omega V \sin\phi$$

In which m is mass of the air parcel. And ω is angular velocity of rotating earth. The angular velocity: $\omega = \frac{2\pi}{T} = \frac{2\times 3.14}{24\times 60\times 60} = 7.275 \times 10^{-5} \frac{rad}{\text{sec}}$ or $\omega = \frac{360}{T} = 4.16 \times 10^{-3}\, degree/sec$. Here $\vec{V}$ is wind velocity. And ϕ is latitude of the place.

At equator $\phi = 0$, hence magnitude of Coriolis force is zero. Coriolis force is zero on equator. At both poles: $\phi = 90^0$, $\sin 90 = 1$ hence Coriolis force is maximum at the North and South poles. When air is at rest, $V = 0$, the Coriolis force acting on it is zero. When air is at rest, no Coriolis force acts on it.

Consider the Earth rotating about its axis - passing through geographic north and south poles. The axis of rotation of Earth is not vertical but makes an angle of 23.5^0 with the vertical as shown in Fig. (2).

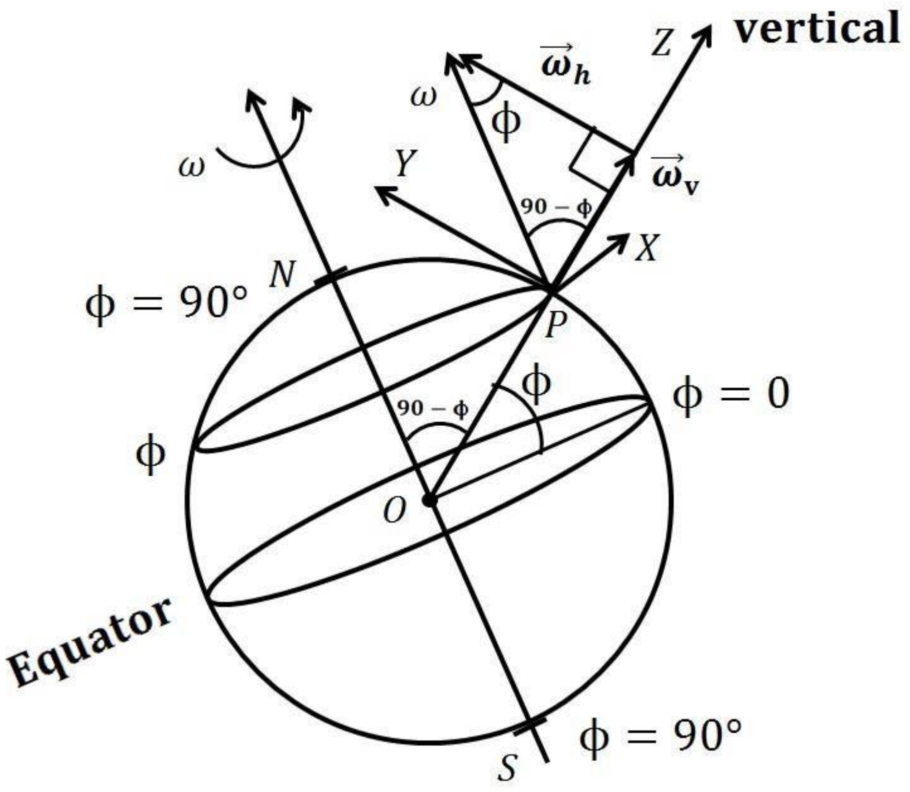

Fig. 2: Coriolis Force

The angular velocity of rotation of Earth is represented by a vector along ON. In which O is centre of the earth and $N -$ its geographic north pole. Consider a place P on the surface of the earth in the north hemisphere. The OP represents position vector of the air mass at P. position vector is a fixed vector or localized vector. A vector having tail at a fixed point is called a fixed or localized vector. e. g. position vector of a moving body is a fixed vector because its tail is fixed at origin. The direction OP when extended outward towards zenith represents vertical at that place (P). If we draw two mutually perpendicular axes viz. X and Y, which are perpendicular to vertical then the plane XY represents horizontal plane at that place. If a body is moving in this horizontal plane with certain velocity $\vec{V}$ then it experiences a force called Coriolis force.

$$\text{The Coriolis force } = -2m(\vec{\omega} \times \vec{V})$$

From figure, the vertical and horizontal components of $\vec{\omega}$ are $\vec{\omega}_V$ and $\vec{\omega}_h$ respectively then by triangle law of vector addition:

$$\vec{\omega} = \vec{\omega}_V + \vec{\omega}_h$$

$$F_C = -2m[(\vec{\omega}_V + \vec{\omega}_h) \times \vec{V}]$$

$$= -2m[(\vec{\omega}_V \times \vec{V}) + (\vec{\omega}_h \times \vec{V})]$$

The angle between $\vec{\omega}_V$ and wind velocity $\vec{V}$ is 90 ° and $\sin 90 = 1$, therefore, first term on R. H. S. exist. And angle between $\vec{\omega}_h$ and wind velocity $\vec{V}$ is 0° and since, $\sin 0 = 0$, therefore, second term on R. H. S. vanishes.

$$\therefore\ F_C = -2m\omega_V V$$

From figure: $\vec{\omega}_V = \omega \sin \phi$ and $\vec{\omega}_h = \omega \cos \phi$

$$\therefore F_C = -2m\omega \sin \phi\, V$$

The direction of Coriolis force acting on wind is given by minus $(\vec{\omega}_V \times \vec{V})$. The Coriolis force acts always at right angles to the direction of air mass; to the right in north hemisphere and to the left in south hemisphere. And this can be determined from equation: $F_C = -2m(\vec{\omega}_V \times \vec{V})$. It is determined first by determining the cross product $(\vec{\omega}_V \times \vec{V})$ and then negating it. The direction of $(\vec{\omega}_V \times \vec{V})$ is determined by right hand rule. Note that: in south hemisphere $\vec{\omega}_V$ is pointing towards the Earth's surface i. e. downward.

Coriolis force deflects the moving air mass towards its right causing anticyclic cyclones in north hemisphere and cyclic cyclones in north hemisphere. And Coriolis force deflects the moving air mass towards its left causing cyclic cyclones in south hemisphere.

The Coriolis parameter is given by

$$f = 2\,\omega\, \sin(\phi)$$

The Coriolis parameter depends upon latitude of the place. Following table gives Coriolis parameter (f) at different latitudes:

Formula $f = 2\,\omega \sin(\phi)$ in which, $\omega = \frac{2\pi}{T} = \frac{2\times3.142}{(24\times60\times60)} = 7.275 \times 10^{-5}\ [rad/s]$

Latitude (ϕ) [deg]	0^o	10^o	20^o	43^o	90^o
Coriolis parameter (f) 10^{-4} [rad/s]	0	0.25	0.50	1	1.458

4. Effects of Coriolis Force in Nature

Coriolis force deflects the moving air mass towards its right causing anticyclic cyclones in north hemisphere. And Coriolis force deflects the moving air mass towards its left causing cyclic cyclones in south hemisphere. This Coriolis force arises when air mass moves or wind blows on the Earth's surface.

Coriolis force causes: i) formation of cyclones,
ii) deflection of river flow,
iii) deflection of missiles and
iv) deflection of freely falling body.

First two are caused by vertical component of Coriolis force while last two are caused by horizontal component of Coriolis force.

Formation of cyclones:

The effect of Coriolis force is that it deflects the moving object towards its right causing anticyclic cyclones in the north hemisphere. And it deflects the moving object towards its left causing cyclic cyclones in the south hemisphere. Coriolis force deflects a freely falling body to the East in both hemispheres. For deflection of freely falling body the horizontal component of Coriolis force is responsible.

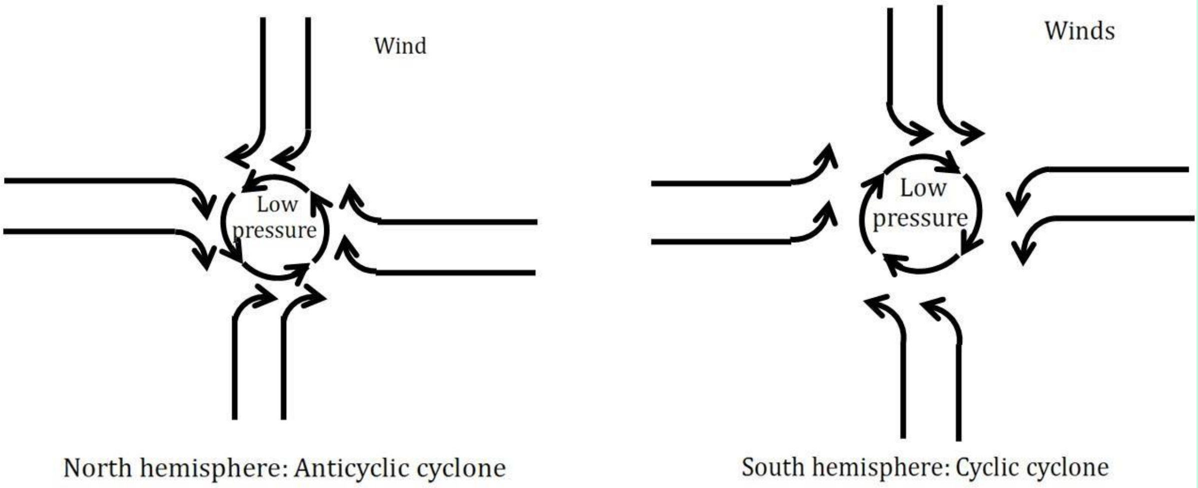

Fig. 3: Formation of Cyclones

ii) Deflection of river flow:

When river flows there is a small component of gravitational force in the forward direction, i. e. in the direction of flow. In this situation the Coriolis force acts across the river flow. It tends to change the direction of flow in the direction of force. The force given by $-2m\omega V \sin \phi$ is independent of θ; angle made by river flow with the X-axis. This component of river flow is independent of direction of river flow.

Thus, the river deviation occurs to the right of river flow in the north hemisphere and to the left of flow in the south hemisphere.

This effect also tends to raise the right banks of rivers in the north hemisphere as compared to the left banks and opposite in the case of south hemisphere.

5. Summary

Inertial frame of reference is one in which Newtons laws hold good. An accelerated or rotating frame of reference is a non-inertial frame of reference. Pseudo forces arise in non-inertial frame of reference. Centrifugal and Coriolis force are pseudo forces.

The Coriolis force causes deflection of missiles. Deflection of river flow. Formation of cyclones. Deflection of freely falling body.

Exercise:

What do we understand?

Quiz on what we learned: Multiple Choice Questions

1) The inertial frame of reference is ....frame of reference.
a) an accelerated b) a rotating
c) unaccelerated d) Both a and c

2) The latitude of the equator is ……
a) 0^0 b) 23.5^0
c) 45^0 d) 90^0

3) A line on map joining places of equal pressure is known as ………
a) Isobars b) Isotherms
c) Isentropic d) Isotach

4) The spatial variation in in pressure is generally called as …………
a) pseudo force b) pressure gradient
c) velocity gradient d) Coriolis force

5) A frame of reference rotating with respect to the fixed frame is known as ……… frame of reference.
a) an inertial b) non-inertial
c) fixed d) pseudo

6) Which of the following force occurs in non-inertial frames of reference?
a) Pressure gradient b) Coriolis
c) Frictional d) Gravitational

7) Centrifugal force is given by $F = \cdots$
a) $-mr\omega^2$ b) $-2m\omega V \sin\phi$
c) $-2m(\vec{\omega} \times \vec{V})$ d) $-\frac{1}{\rho}\frac{dp}{dl}$

8) is a pseudo or fictitious force.
a) pressure gradient force b) frictional force
c) centrifugal force d) gravity force

9) In case of the earth, the centrifugal acceleration has the maximum value at the ….
a) north pole b) equator
c) south pole d) line of Capricorn

10) The earth is bulged at the equator and flattened at the poles due to …...
a) Coriolis force b) frictional force
c) centrifugal force d) frictional force

Answers

1) c) unaccelerated 2) a) 0^0 3) a) Isobars 4) b) pressure gradient
5) b) non-inertial 6) b) Coriolis 7) a) $-mr\omega^2$ 8) c) centrifugal force
9) b) equator 10) c) centrifugal force

11) On the earth's surface centrifugal force is maximum at latitude ……..
a) $\phi = 0°$ b) $\phi = 30°$
c) $\phi = 45°$ d) $\phi = 90°$

12) The Coriolis force acting on a moving air mass is due to …. of the Earth.
a) rotation b) revolution
c) tilt of the axis d) none of the above

13) The Coriolis force is given by $F_c = \cdots$
a) $-2m\vec{\omega} \times \vec{V}$ b) ma
c) $-2m\vec{\omega} \times \vec{\omega} \times \vec{r}$ d) $mr\omega^2$

14) At the equator $(\phi = 0)$ the Coriolis force is ………
a) minimum b) zero
c) maximum d) moderate

15) At both Poles, the magnitude of Coriolis force acting on the moving air mass is ….
a) minimum b) zero
c) moderate d) maximum

16) Coriolis acceleration is given by ……
a) $2\vec{\omega} \times \left(\frac{d\vec{r}}{dt}\right)_{rot}$ b) $2\vec{\omega} \times \left(\frac{d\dot{\vec{r}}}{dt}\right)_{rot}$
c) $2\vec{\omega} \times \left(\frac{d\ddot{\vec{r}}}{dt}\right)_{rot}$ d) $2\vec{\omega} \times \left(\frac{d\vec{V}}{dt}\right)_{rot}$

17) If air mass is at rest $(V = 0)$ then Coriolis force acting on it is ……
a) zero b) moderate
c) maximum d) minimum

18) Which of the following statement is correct?
a) Coriolis force tends to raise the right banks of rives in the south hemisphere.
b) Coriolis force tends to raise the left banks of rivers in the north hemisphere.
c) The component of coriolis force which deviates river flow is $2\omega v \cos\phi$.
d) The component of coriois force acting on river flow is independent of direction of flow.

19) The Coriolis parameter is given by $f = \cdots$
a) $2\omega \sin\phi$ b) $2\omega \cos\phi$
c) $2\omega \tan\phi$ d) $2\omega \cot\phi$

20) The angular velocity (ω) of the Earth is equal to ……
a) $4.16 \times 10^{-3}\ rad/sec$ b) $7.275 \times 10^{-5}\ rad/sec$
c) $7.275 \times 10^{-5}\ degree/sec$ d) $4.16 \times 10^{-3}\ degree/sec$

21) Generally, air flows from a region of … pressure to ……pressure.
a) high, low b) low, high
c) low, low d) high, high

Answers:

11) a) $\phi = 0°$ 12) a) rotation 13) a) $-2m\vec{\omega} \times \vec{V}$ 14) b) zero
15) d) maximum 16) a) $2\vec{\omega} \times \left(\frac{d\vec{r}}{dt}\right)_{rot}$ 17) a) zero
18) d) The component of Coriolis force acting on river flow is independent of direction of flow.
19) a) $2\omega \sin\phi$ 20 b) $7.275 \times 10^{-5}\ rad/sec$ 21) a) high, low

Feedback: (Tick Mark Your Option)

1) How was the learning experience?

Outstanding/Excellent / Nice /Good/Fair

2) Which aspect do you like most?

Introduction/Concept/Diagrams/Exercise/Applications

3) Anything else to be added?

Applications/Problems/Illustrations/Notes/Videos

4) Which point was not up to the mark and need revision?

Introduction/Concept/Diagrams/Exercise/Applications

5) Suggestions if any:

....

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Name of the student and class:

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References and Know More:

1) CLASSICAL MECHANICS (2nd) – HERBERT GOLDSTEIN (Narosa)
2) INTRODUCTION TO Classical Mechanics – R G TAKWALE, P S PURANIK (Tata McGraw-Hill)
3) CLASSICAL MECHANICS – GUPTA, KUMAR, SHARMA (PRAGATI)

Ctrl+Click and follow the links to know more

4) Talk on Centrifugal Force - YouTube
5) HC Verma Kinematics 3 of 6 Deflects things sideways Coriolis Force - YouTube
6) W1 01 Inertial and non intertial frames | HC VERMA BSC LECTURES | GDS K S - YouTube

Tell Me and I Forget.

Teach Me and I May Remember.

Involve Me and I Learn. - **Benjamin Franklin**

HAVE A NICE LEARNING EXPERIENCE

~ Thank you ~