Subject	**Physics**
Paper Number and Title	**VIII: Modern Physics**
Module Number and Title	**1: Theory of Relativity**
Module Tag	**DAYA_PHY_CVC_M1**

Dr. Chetan Chanmal

Assistant Professor

Department of Physics

D. B. F. Dayanand College of Arts & Science, Solapur

PHYSICS	**Paper VIII: Modern Physics**
	Module 1: Theory of Relativity

Objectives:

1. To Study Theory of Relativity
2. Galilean Transformation Equation
3. Mass Energy Relation
4. Twin Paradox

Outcomes:

1. Understand the concepts in theory of relativity
2. Understand the Galilean transformation equation
3. Understand the conversion of mass and energy
4. Understand the concept of Twin Paradox

Table of Content:

1. Introduction:

The special theory of relativity was formulated by Albert Einstein in 1905, which deals with problems related to inertial frames of references. Then in the year 1917, Einstein developed the 'General theory of relativity' which deals with problems related to non-inertial frames of reference. We now study only 'special theory of relativity'. Here we study how the measurements depend upon the observer as well as upon what is observed. A new mechanics is emerged from relativity in which there are exact relationships between space and time, mass and energy. With these relationships, it is possible to understand the microscopic world within the atom, explanation of which is the central problem of modern physics.

Frame of reference and an event:

In order to describe the state of a body, one requires a system with respect to which the position of the body with the time can be measured. Such a system is called as frame of reference. Thus a frame of reference is a co-ordinate system attached to a rigid body to describe the relative position of a body or any particle in space. The simplest frame of reference is cartesian coordinate system.

In physics, a point is defined as the position in space where a physical phenomenon is supposed to be occurred. The time of occurrence of a physical phenomenon together with the point is called an event. Thus an event is described with four co-ordinate system or space-time co-ordinate system as shown in Fig. 1.1.

An inertial frame of reference is the frame of reference in which Newton's laws of motion are valid. It is also called as unaccelerated frame of reference. It may be a body which will be acted upon by zero external force and hence moving with constant velocity. Thus a frame of reference moving with constant velocity relative to fixed frame is called as inertial frame of reference.

The special theory of relativity deals with the description of events by observers in the inertial frames of reference. Reference frame attached to the earth can be considered as an inertial frame of reference for most practical purposes. Similarly, reference frames moving with uniform velocity with respect to each other and with respect to fixed stars are examples of inertial frames.

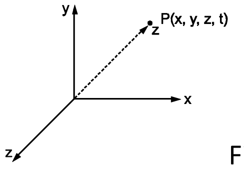

Fig 1.1

2. Galilean transformation equation:

Galileo classically formulated the principle of relativity, which is known as the Galilean relativity. The algebraic equations connecting the two sets of co-ordinates and time are called transformation equations.

Consider two inertial frames of reference S and S'. Let S be stationary and S' be moving with uniform velocity v along positive X-direction. For convenience, choose the three sets of axes to be parallel and their relative motion be along the common X, X' axis. (Refer Fig. 2.2)

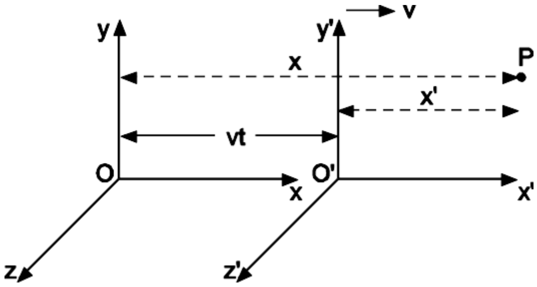

Fig. 1.2

Let an event occur at point P. Let x, y, z and t be the space and time co-ordinates of an event as measured in S and x', y', z' and t' be the co-ordinates of this event as measured in S'.

Let the origins O and O' of frames S and S' coincide at t = t' = 0. Then, following equations give the transformation of the co-ordinates of space and time from S to S'.

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ \text{and} \quad t' &= t \end{aligned}\right\} \quad \ldots (1.1)$$

These equations are called Galilean transformation equations. To transform the co-ordinates from frame S' to S, the velocity v is replaced by – v and the exchanging primed by unprimed quantities and vice-versa, the transformation equations become

$$\left.\begin{aligned} x &= x' + vt' \\ y &= y' \\ z &= z' \\ \text{and} \quad t &= t' \end{aligned}\right\} \quad \ldots (1.2)$$

From these transformation equations, we conclude that the basic quantities in mechanics viz. length (space), time etc. are independent of its motion with respect to an observer. In order words, these are absolute quantities.

3. Mass Energy Relation

The mass-energy relation is most fascinating and important result of special theory of relativity. According to this theory, mass and energy are equivalent quantities.

According to Newton's second law of motion, the force is a rate of change of momentum i.e.

$$F = \frac{d}{dt}(mv)$$

But according to theory of relativity, both mass and velocity are variables.

$$\therefore \qquad F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

Suppose the force F acts through a distance dx, so increase in the kinetic energy of the body is dT given by

$$dT = F\,dx = m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$$

$$= m\frac{dx}{dt}dv + v\frac{dx}{dt}dm$$

But $\frac{dx}{dt}$ is the velocity v.

$$\therefore \qquad dT = mv\,dv + v^2\,dm \qquad \ldots (1.39)$$

Now, the variation of mass with velocity is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring both the sides, we have

$$m^2 = \frac{m_0^2 c^2}{c^2 - v^2}$$

or $\quad m^2c^2 - m^2v^2 = m_0^2c^2$

Since c and m_0 are constant, while m and v are variables, differentiating above equation, we get

$$c^2 \cdot 2m\,dm - v^2 \cdot 2m\,dm - m^2 \cdot 2v\,dv = 0$$

or $\;c^2\,dm - v^2\,dm - mv\,dv = 0$

$$\therefore \quad mv\,dv + v^2\,dm = c^2\,dm \qquad \ldots (1.40)$$

Comparing equations (1.39) and (1.40), we have

$$dT = c^2\,dm \qquad \ldots (1.41)$$

When a body is accelerated from rest to a velocity v, its mass increases from the rest mass m_0 to m and the total kinetic energy required is obtained by integration of equation (1.41) as

$$T = \int_{m_0}^{m} c^2 \, dm$$

or $$T = (m - m_0) c^2$$

Above equation states that K.E. of a body is equal to the increase in its mass caused due to its relative motion multiplied by square of the speed of light. Above equation can be rewritten as

$$mc^2 = T + m_0c^2$$

where mc^2 is the total energy E of the body.

If body is at rest and T = 0, the body possesses the energy m_0c^2 and is called rest energy E_0 of a body whose mass at rest is m_0.

Thus, $$E = E_0 + T$$

where $$E_0 = m_0c^2$$

and $$E = mc^2$$

This is Einstein's mass-energy relation which states a universal equivalence between mass and energy.

4. Twin Paradox

One of the most fascinating paradox in the theory of relativity is twin paradox or clock paradox. This paradox involves two identical clocks. Let these clocks are replaced with a pair of identical twins named as A and B.

This paradox is stated as follows:

Two identical twins A and B are at rest on the earth. One of them say B goes on space voyage by rapidly moving rocket and returns back after sufficient number of years. The twin B surprised to see that 'A' has grown older while A finds 'B' is remaining pretty young as compared to him.

Now the paradox is this why does not happen other way round due to the relative motion of A with B. That is after return journey B, can think it was he who remained stationary and that A went on the space voyage. This should result in younger appearance of A than B. But it is not true.

The resolution of the paradox depends upon the fact that there is no symmetry between B and A. The B changes the frame of reference when A remains the same frame. Further B along with rocket has undergone acceleration at various times of its journey when he takes off, when he turns round and when he finally comes to stop. Therefore, the formula of special theory of relativity, which hold only for frames of reference in relative motion at constant velocity, cannot be applied in this situation. It is therefore, B, the space traveler, is indeed younger on his return journey than his twin brother A. In this way, clock paradox or twin paradox is get resolved.

Summary

1. The Special Theory of Relativity is based on variation of length, mass and time with velocity.

2. The concept of Inertial and Non-inertial frame of references were understood.

3. Galilean transformation equation help in transforming from one inertial frame into another inertial frame of references.

4. The energy mass relation is derived from postulates of special theory of relativity.

References and Know More

Video 1:

https://www.youtube.com/watch?v=ttZCKAMpcAo

Video 2:

https://www.youtube.com/watch?v=TgH9KXEQ0YU

Video 3:

https://www.youtube.com/watch?v=Ie0Mllx4njA

Know More

Suggested References and weblink

1. https://en.wikipedia.org/wiki/Theory_of_relativity

2. https://www.nationalgeographic.com/science/article/einstein-relativity-thought-experiment-train-lightning-genius

Assignment

Quiz on what we learned: Multiple Choice Question.

1. The special theory of relativity was developed by ……
   (a) Einstein (b) Newton
   (c) Galileo (d) Lorentz
2. The inertial frame of reference is …… frame of reference.
   (a) an accelerated (b) unaccelerated
   (c) a rotating (d) a steady
3. Who formulated first the classical theory of relativity?
   (a) Einstein (b) Newton
   (c) Galileo (d) Lorentz
4. Galilean transformation for x coordinate from S to S' is ……
   (a) $x' = x - vt$ (b) $x' = x - \frac{vt}{c^2}$
   (c) $x' = \frac{x - vt}{c^2}$ (d) $x' = x - vc^2$
5. According to Einstein, velocity of light in free space is ……
   (a) dependent on the direction of propagation
   (b) variable
   (c) a constant
   (d) dependent on motion of source
6. The length of rod or object in motion with the velocity nearly equal to c ……
   (a) remains the same (b) decreases
   (c) increases (d) becomes zero
7. The time of moving clock with velocity comparable to the velocity of light ……
   (a) increases (b) decreases
   (c) remains constant (d) becomes zero
8. Mass-energy equivalence is ……
   (a) E = mc (b) E2 = mc2
   (c) E = mc2 (d) E2 = m2c
9. In twin paradox effect, the travelling twin is ......
   (a) younger (b) lighter
   (c) older (d) longer
10. Mass of moving object always ……
   (a) increases (b) decreases
   (c) remains the same (d) be zero

Answers:

1. (a)	**2. (b)**	**3. (c)**	**4. (a)**	**5. (c)**
6. (b)	**7. (a)**	**8. (c)**	**9. (a)**	**10. (a)**

Feedback

1) How was the learning experience?

Outstanding/Excellent / Nice /Good/Fair

2) Which aspect do you like most?

Introduction/Concept/Diagrams/Exercise/Applications

3) Anything else to be added?

Applications/Problems/Illustrations/Notes/Videos

4) Which point was not up to the mark and need revision?

Introduction/Concept/Diagrams/Exercise/Applications

5) Suggestions if any:

....

....

Name of the student and class:

____

____